POINT CLOUD SAMPLING AND RESAMPLING STRATEGIES

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Objectives

- Establish a general method for down-selecting 3D LiDAR data to desired product sampling
- Enable reduced samples per product without impacting achievable resolution
- Efficiently encode Levels of Detail
- Leverage sampling strategies for GmAPD product generation

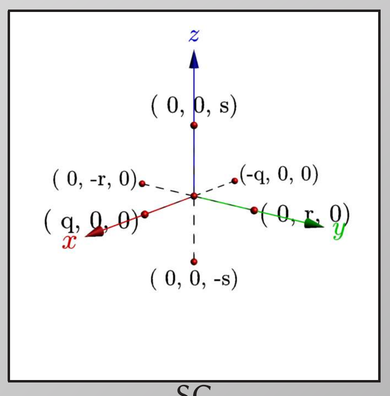
Sampling Lattices

- The Petersen-Middleton theorem [1] informs selection of optimal sampling lattices for n-dimensional fields to an objective wave-number limit.
- Three lattices appear in the context of optimal sampling in $\mathbb{R}^3$: the SC lattice, the FCC lattice, and the BCC lattice.
- The SC lattice is often considered to be a natural choice since it establishes a Cartesian coordinate system.

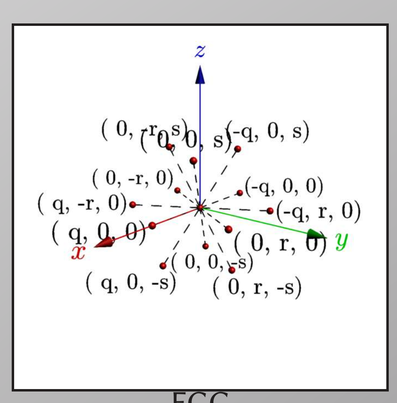
Basis Selection

- Fundamental sampling matrices are derived from SC, FCC, and BCC generating matrices that are normalized and rotated so that two of the basis vectors span the xy-plane:

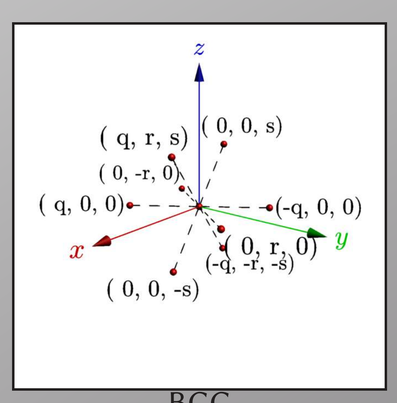
$$\boldsymbol{B}_{\text{SC}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{B}_{\text{FCC}} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{6}}{3} \end{bmatrix} \qquad \boldsymbol{B}_{\text{BCC}} = \begin{bmatrix} \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{3} & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \\ 0 & 0 & \frac{\sqrt{6}}{3} \end{bmatrix}$$



SC



FCC



BCC

Sample Interval Scale Factors

- Sampling intervals, r_h and r_v, are set by scaling the fundamental matrices by

$$\boldsymbol{S} = \begin{bmatrix} r_{\text{h}} & 0 & 0 \\ 0 & r_{\text{h}} & 0 \\ 0 & 0 & \frac{1}{b_{3,3}} r_{\text{v}} \end{bmatrix}$$

- Uniform scaling of the sampling lattice for a given horizontal sampling interval is given by setting $r_{\text{v}} = b_{3,3} r_{\text{h}}$.

Characteristic-Preserving Scale Factors

- Specific characteristics can be maintained between lattice types by setting scale appropriately

Feature Preserving Scale Factors for Sampling Lattices

Preservation Strategy	SC	FCC	BCC
Spacing (s_i)	1	1	1
Density (s_d)	1	$\frac{\sqrt[4]{108}}{3}$	$\frac{\sqrt[4]{18}}{2}$
Samples (s_p)	1	$\sqrt[6]{2}$	$\frac{\sqrt[6]{108}}{2}$
Texture Resolution (s_t)	1	$\frac{2\sqrt{3}}{3}$	$\frac{3\sqrt{2}}{4}$
Spatial Resolution (s_s)	1	$\frac{\sqrt{6}}{2}$	$\frac{\sqrt{6}}{2}$

- The resulting lattice site densities reveal which lattices are optimal for each constraint.

Discrete Lattice Samples Relative to SC Lattice ($\Delta\%$)

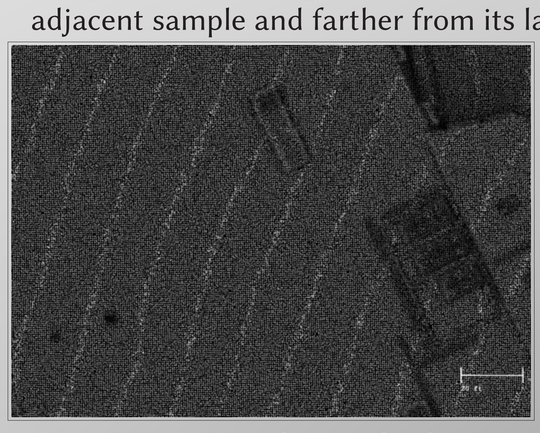
	3-D		2-D	
Preservation Strategy	FCC	BCC	FCC	BCC
Spacing	+41.42	+29.90	+15.47	+6.07
Density	+13.98	+18.92	—	—
Samples	—	—	−8.35	−10.91
Texture Resolution	−8.14	+8.87	−13.40	−5.72
Spatial Resolution	−23.02	−29.29	−23.02	−29.29

Sampling Strategies

- Point cloud sampling performed by stratified clustered sampling
- Lattice sites define strata
- Two clustering mechanisms recommended: Nearest-Center and Mass-Point

Sample Aliasing and Mitigation

- Requiring a sample for each occupied lattice cell causes aliasing.
- Mitigate by removing samples that are closer than half a sample interval to an adjacent sample and farther from its lattice site than its neighbor.



Near-Center Sampling with Aliasing

Near-Center Sampling with Mitigation

Level of Detail Generation and Encoding

- LODs are generated by identifying sampled points for each LOD lattice constraint.
- LODs are represented by point duplication (separate files, file regions) or point attribution (bit mask per point, index files).
- Probability of point selection for an LOD can be approximated by the relative sampling densities of the LOD sites in the xy-plane and base point cloud ANPD

$$\Pr(\mathcal{L}) = \Pr(p \in \mathcal{L}) = \frac{\rho_{\mathcal{L}}}{\rho_{\text{nom}}}$$

- Estimated number of points for respective strategies:

Duplication

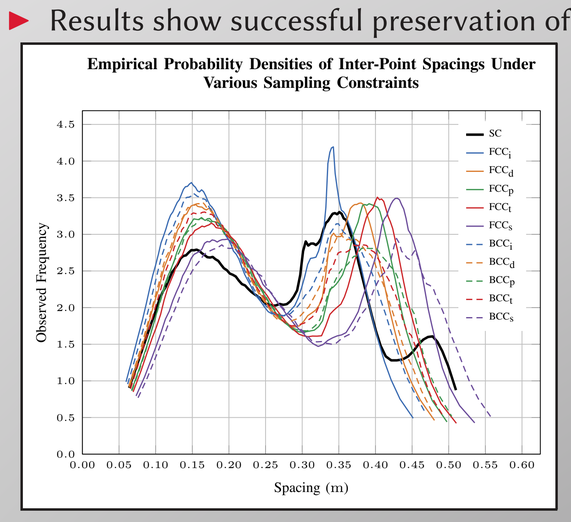
$$c_{\text{dup}} = n \sum_{i=1}^{L} \Pr(\mathcal{L}_i)$$

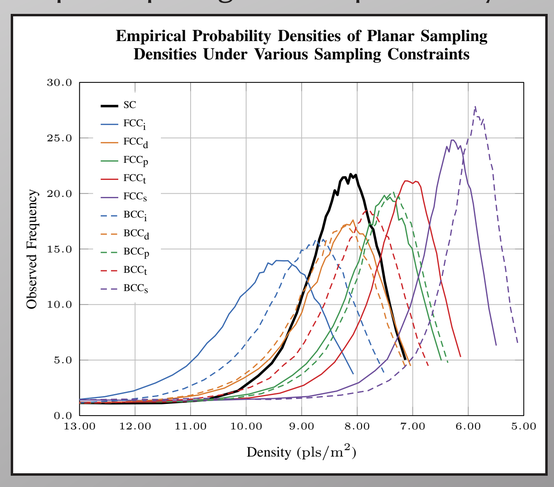
Attribution

$$c_{\text{att}} = n \left(1 - \prod_{i=1}^{L} (1 - \Pr(\mathcal{L}_i)) \right)$$

Characteristic-Preserving Scale Factor Results

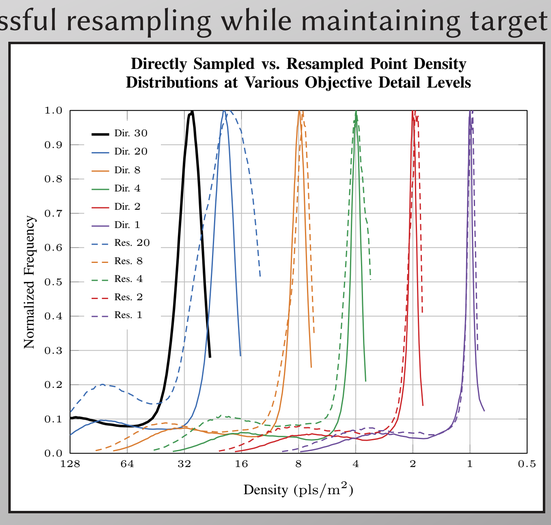
- GmAPD product formed for each lattice type to preserve characteristics of 8 pls m^{-2} SC.
- Density and spacing were analyzed by TIN/Voronoi Diagram method. [2]
- Results show successful preservation of inter-point spacing and sample density.





LOD Resampling Results

- Point Clouds formed at 30, 20, 8, 4, 2, and 1 pls/m^2 with FCC density preservation
- LODs generated from 30 pls/m^2 base data at same target densities
- Results show successful resampling while maintaining target density.



Conclusions and Discussion

- Developed lattice-constrained stratified sampling approach
- Leveraged stratified sampling for LOD generation
- Established conditions for determining optimal LOD representation
- Demonstrated efficient sampling to preserve desired characteristics

References

[1] Daniel P. Petersen and David Middleton, "Sampling and Reconstruction of Wave-Number–Limited Functions in N-Dimensional Euclidean Spaces," *Information and Control*, vol. 5, no. 4, pp. 279–323, 1962.

[2] Ty Naus, "Unbiased LiDAR Data Measurement (Draft)," 2009.